Sincronización y afinación

Dos ejemplos de sistemas oscilantes colectivos en interacción

Pablo Bolcatto

9 de junio de 2017

Seminario del IMAL "Carlos Segovia Fernandez"

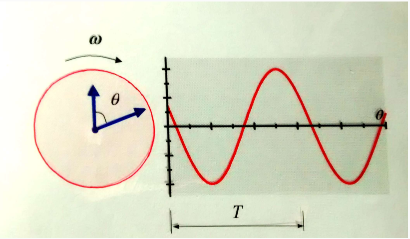
Esquema de la charla

1. Motivación
2. Modelo de sincronización
(Kuramoto, 1975)
3. Modelo de afinación
(Urteaga-Bolcatto, 2003)
4. Discusión abierta

Motivación

Motivación

Oscilador armónico simple



$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

Motivación

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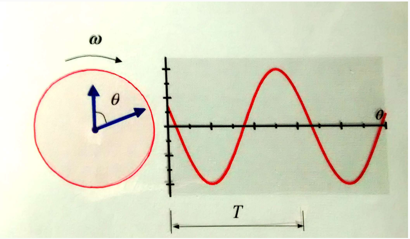

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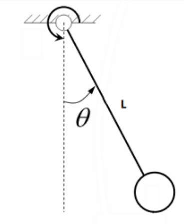
Péndulo simple



$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

Motivación

Christiaan Huygens (La Haya, 1629-1695).
Astrónomo, físico y matemático de los Países Bajos.

Ejemplos

Animados

Luciérnagas de Kuala Lumpur

Inanimados

Sincronización de metrónomos

Otros ejemplos

- ✔ Aplausos
- ✔ Marcapasos
- ✔ Neuronas
- ✔ Junturas Josephson
- ✔ Mujeres convivientes
- ✔ Etc., etc., etc.

Modelo de sincronización (Kuramoto, 1975)

Modelo de Kuramoto

Lecture Notes in Physics

Edited by J. Ehlers, München, K. Hepp, Zürich, and H. A. Weidenmüller, Heidelberg
Managing Editor: W. Beiglböck, Heidelberg

39

International Symposium on Mathematical Problems in Theoretical Physics
January 23–29, 1975, Kyoto University, Kyoto/Japan

Edited by H. Araki

Springer-Verlag
Berlin · Heidelberg · New York 1975

SELF-ENTRAINMENT OF A POPULATION OF COUPLED NON-LINEAR OSCILLATORS

Yoshiki Kuramoto
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Temporal organization of matter is a widespread phenomenon over a macroscopic world in far from thermodynamic equilibrium. A previous study on chemical instability[1] implies that a simplest nontrivial model for a temporally organized system may be represented by a macroscopic self-sustained oscillator Q obeying the equation of motion

$$\dot{Q} = (i\omega + \alpha)Q - \beta|Q|^2 Q\,, \quad \alpha,\beta > 0. \tag{1}$$

Consider a population of such oscillators $Q_1, Q_2, \cdots Q_N$ with various frequencies, and introduce interactions between every pair as follows.

$$\dot{Q}_s = (i\omega_s + \alpha)Q_s + \sum_{r\neq s} v_{rs}Q_r - \beta|Q_s|^2 Q_s\,, \quad r,s = 1,2,\cdots N\,. \tag{2}$$

We found that it is possible to construct from (2) a soluble model for a community exhibiting mutual synchronization or self-entrainment above a certain threshold value of the coupling strength. Such a type of phase transition has been considered by Winfree[2] without resorting to specialized models but only phenomenologically.

Our simplifying assumptions are:

(I) $v_{rs} = v/N$ independently of r and s,
(II) $\alpha,\beta \to \infty$ but α/β, ω_s, v = finite,
(III) $N \to \infty$.

Let us put $Q_s = \rho_s e^{i\varphi_s}$. Owing to the assumption (II), the amplitude ρ_s may be fixed at $\sqrt{\alpha/\beta}$. Thus we have only to consider the equation

$$\dot{\varphi}_s = \omega_s + \frac{v}{N}\sum_r \sin(\varphi_r - \varphi_s)\,. \tag{3}$$

As an illustration, we summarize the results obtained when the distribution of the native frequency is a Lorentzian with the peak at ω_0 and the width γ. In this case the threshold condition is

$$\eta \equiv 2|\gamma/v| = 1\,. \tag{4}$$

Modelo de Kuramoto

Un oscilador $\dot{\theta} = \omega$

Modelo de Kuramoto

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N osciladores $\dot{\theta}_i = \omega_i \qquad i = 1, ..., N$

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N osciladores acoplados $\dot{\theta}_i = \omega_i - \sum_{j=1}^{N} K_{ij} \sin(\theta_i - \theta_j + \alpha) \qquad i, j = 1, ..., N |\alpha| \leq \pi/2$

Modelo de Kuramoto

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$$\dot{\theta}_i = \omega_i - \frac{K}{N} \sum_{j=1}^{N} \sin(\theta_i - \theta_j + \alpha) \quad \leftarrow \text{acoplamiento constante}$$

Modelo de Kuramoto

$$\boxed{\dot{\theta}_i = \omega_i - \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_i - \theta_j + \alpha)}$$

Modelo de Kuramoto

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Definimos un parámetro de orden

$$\sigma e^{\imath\Omega t} = \frac{1}{N}\sum_{j=1}^{N} e^{\imath\theta_j} \quad \rightarrow \quad \sigma\cos\Omega t = \frac{1}{N}\sum_{j=1}^{N}\cos\theta_j \,;\; \sigma\sin\Omega t = \frac{1}{N}\sum_{j=1}^{N}\sin\theta_j$$

Modelo de Kuramoto

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$$\boxed{\begin{aligned} \dot{\theta}_i &= \omega_i - \tfrac{K}{N}\textstyle\sum_j \sin(\theta_i - \theta_j + \alpha) \\ \dot{\theta}_i &= \omega_i - K\sigma \sin(\theta_i - \Omega t + \alpha) \end{aligned}} \; i(j) = 1, ...N$$

Modelo de Kuramoto

Veamos casos límites:

$$\dot{\theta}_i = \omega_i - K\sigma \sin(\theta_i - \Omega t + \alpha) \quad ; \quad \sigma e^{\imath \Omega t} = \frac{1}{N}\sum_{j=1}^{N} e^{\imath \theta_j}$$

- Sincronización nula $\rightarrow \dot{\theta}_i = \omega_i \rightarrow \boxed{\sigma = 0}$

Modelo de Kuramoto

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- Sincronización completa $\rightarrow \theta_i = \theta_j \quad \forall \quad i, j \rightarrow \boxed{\sigma = 1}$

Modelo de Kuramoto

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- Sincronización completa $\rightarrow \theta_i = \theta_j \quad \forall \quad i, j \rightarrow \boxed{\sigma = 1}$
- Sincronización parcial $\boxed{0 < \sigma < 1}$

Modelo de Kuramoto

Progress of Theoretical Physics, Vol. 76, No. 3, September 1986

A Soluble Active Rotator Model
Showing Phase Transitions via Mutual Entrainment

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(Received April 17, 1986)

Some analytical results are obtained for a large population of limit-cycle oscillators modelled by a set of deterministic equations $\dot{\phi}_i = \omega_i - N^{-1}K\sum_{j=1}^{N}\sin(\phi_i - \phi_j + \alpha)$ $(i=1, 2, \cdots, N)$, where ϕ_i is the phase of the i-th oscillator and ω_i's are parameters distributed randomly. The present work is a generalization of the previous one where the study was limited to the case of vanishing α and symmetric distribution of ω_i. As in the previous case, a particular macroscopic solution of steady rotation is found, which branches off the trivial solution at some positive K. A computer simulation with $N=1000$ is carried out, which correctly reproduces our analytical results.

§ 1. Introduction and model equations

Large populations of coupled limit-cycle oscillators are known to exhibit many interesting behaviors through mutual synchronization.[1)~14)] A convenient mathematical model for studying such systems is given by a set of differential equations,[2)~4)]

$$\dot{\phi}_i = \omega_i - \sum_{j=1}^{N} K_{ij}\sin(\phi_i - \phi_j + \alpha), \qquad |\alpha| \le \frac{\pi}{2}, \tag{1}$$

where ϕ_i represents the phase of the i-th oscillator, and N the total number of the oscillators. The natural frequencies ω_i may change randomly from oscillator to oscillator, but they are fixed in time; the normalized number density of the oscillators having natural frequency ω is denoted as $g(\omega)$. Let us consider the case of *uniform coupling*[2)~4)] for which our model is simplified to

$$\dot{\phi}_i = \omega_i - N^{-1}K\sum_{j=1}^{N}\sin(\phi_i - \phi_j + \alpha)\,. \tag{2}$$

In this special case, analytical expressions for various quantities can be obtained. The most important of them is the complex order parameter defined by

$$\sigma\,\exp(i\theta) = N^{-1}\sum_{j=1}^{N}\exp(i\phi_j)\,. \tag{3}$$

Modelo de Kuramoto

580 *H. Sakaguchi and Y. Kuramoto*

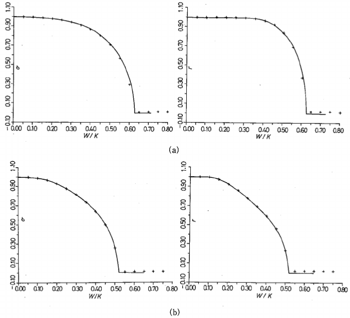

Fig. 1. Order parameters σ and r as a function of W/K. (a) $\alpha=0$, (b) $\alpha=\pi/4$. The solid curves show analytical results whose dependence on W and K is only through W/K. The crosses show the results from our computer simulation in which K was set to 1 throughout.

H. Sakaguchi and Y. Kuramoto

Progress of Theoretical Physics 76 (1986)

N osciladores con distribución gaussiana de frecuencias propias

$$g(\omega) = Ae^{-(\omega-\Omega)^2/W^2}$$

Parámetro de orden alternativo

$$r = \frac{N_s}{N}$$

N_s osciladores sincronizados

Modelo de Kuramoto

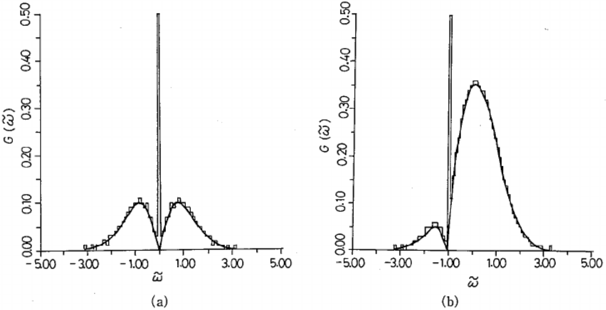

Fig. 2. Distribution $G(\tilde{\omega})$ of the resultant frequencies $\tilde{\omega}$ in the ordered state. (a) $\alpha=0$, $W/K=0.55$, (b) $\alpha=\pi/4$, $W/K=0.50$. The solid curves show analytical results, but delta-peaks that should appear at $\tilde{\omega}=\Omega$ are not indicated. The hitograms show the results from our computer simulation where $\tilde{\omega}_i$ was defined as the long-time average of $d\phi_i/dt$.

H. Sakaguchi and Y. Kuramoto

Progress of Theoretical Physics 76 (1986)

Ejemplo: Aplausos rítmicos

PHYSICAL REVIEW E VOLUME 61, NUMBER 6 JUNE 2000

Physics of the rhythmic applause

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T. Vicsek
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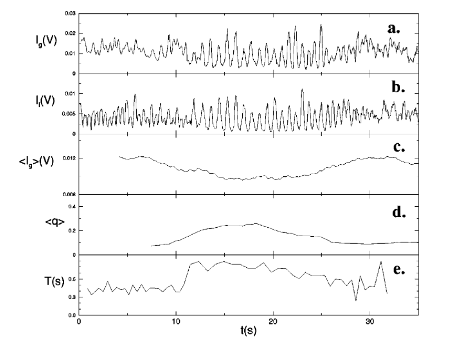
(Received 23 February 2000)

We report on a series of measurements aimed to characterize the development and the dynamics of the rhythmic applause in concert halls. Our results demonstrate that while this process shares many characteristics of other systems that are known to synchronize, it also has features that are unexpected and unaccounted for in many other systems. In particular, we find that the mechanism lying at the heart of the synchronization process is the period doubling of the clapping rhythm. The characteristic interplay between synchronized and unsynchronized regimes during the applause is the result of a frustration in the system. All results are understandable in the framework of the Kuramoto model.

Ejemplo: Aplausos rítmicos

Grabaciones en teatros de Rumanian y Hungría.



(2) An experimentally computable order parameter q_{exp} was calculated. This order parameter was defined in a very analogous way with the q order parameter (3) in the Kuramoto model. At each time step q_{exp} is calculated as the maximum of the normalized correlation between the $s(t)$ signal and a harmonic function

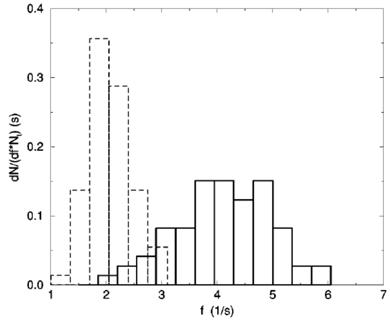
$$q_{exp}(t) = \max_{\{T,\phi\}} \left\{ \frac{\int_{t-T}^{t+T} s(t)\sin(2\pi/T+\phi)dt}{\int_{t-T}^{t+T} s(t)dt} \right\} \quad (5)$$

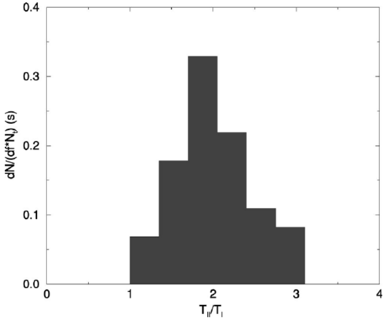
Z. Neda *et el*

Phys. Rev. E 61 (6987)

Ejemplo: Aplausos rítmicos

Experimento controlado: Aplaudidores (rumanos y húngaros) no interactuantes.





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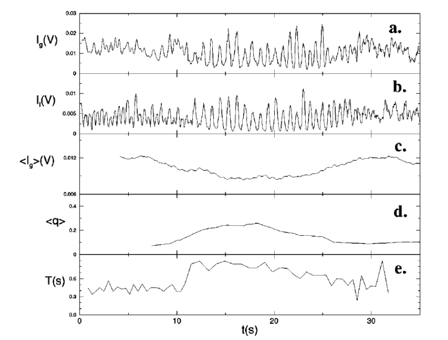
Phys. Rev. E 61 (6987)

Ejemplo: Aplausos rítmicos



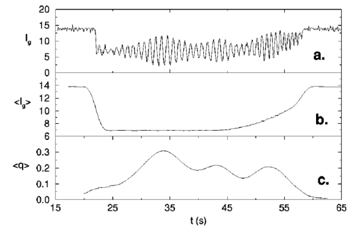
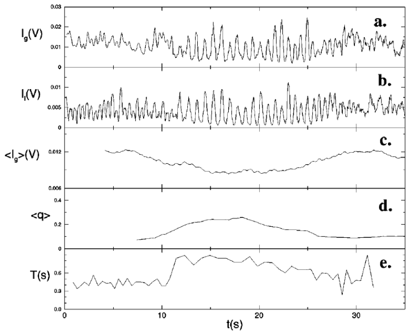
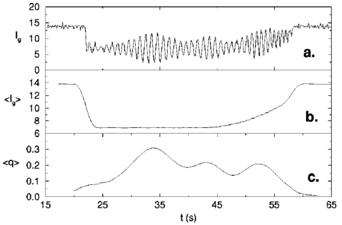


FIG. 6. Computer simulation of the Kuramoto model for $N=70$ rotators ($K=0.8\ \mathrm{s}^{-1}$, $\bar{\omega}=2\pi s^{-1}$, and $D=2\pi/6.9\ \mathrm{s}^{-1}$). We double the rotators periods at $t_1=21$ s and linearly increase the frequency back to the original value from $t_2=35$ s. The noise pulse given by one oscillator has $I_0=\omega/\bar{\omega}$ intensity and $\tau=0.01$ s duration.

Ejemplo: Aplausos rítmicos





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(3) In communist times it was a common habit to applaud by rhythmic applause the ‘‘great leader’’ speech. During this rhythmic applause the synchronization was almost never

lost. This is very nice evidence of the fact that spectators were not enthusiastic enough and were satisfied with the obtained global noise intensity level, having no desire to increase it. Frustration was not present in this system.

Modelo de afinación (Urteaga-Bolcatto, 2003)

Modelo de afinación

Dynamics of the tuning process between singers.

R. Urteaga[1] and P. G. Bolcatto[2]

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Abstract. We present a dynamical model describing a predictable human behavior like the tuning process between singers. The purpose, inspired in physiological and behavioral grounds of human beings, is sensitive to all Fourier spectrum of each sound emitted and it contemplates an asymmetric coupling between individuals. We have recorded several tuning exercises and we have confronted the experimental evidence with the results of the model finding a very well agreement between calculated and experimental sonograms.

PACS. 43.66.Ba Models and theories of auditory processes and 43.75.CdMusic perception and cognition – 43.75.Rs Singing

R. Urteaga and P. G. Bolcatto

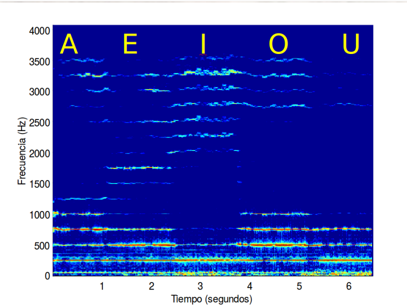
European Journal of Physics (2003)

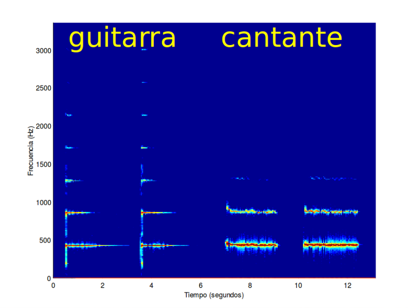
Modelo de afinación. Propuesta

¿Qué es un sonido musical?

- Es una perturbación de un medio material (por ejemplo, aire) compuesta por una familia de frecuencias.
- A la menor frecuencia se la llama "fundamental" y al resto se las denomina "armónicas"
- Las armónicas son múltiplos enteros de la fundamental: $f_o, f_1 = 2f_o, f_2 = 3f_o, ...$
- El número de armónicos y la importancia relativa con la que intervienen en un sonido, determina el timbre del mismo pero no cambia la "nota".

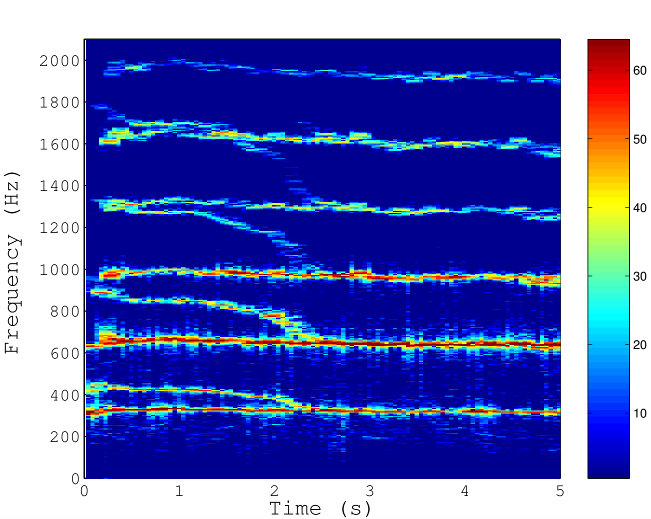
Modelo de afinación. Propuesta





Modelo de afinación. Ejemplos

Consigna: *'Afinar en la misma nota'*



Modelo de afinación. Propuesta

Proponemos la siguiente dinámica para las frecuencias fundamentales de cada emisor:

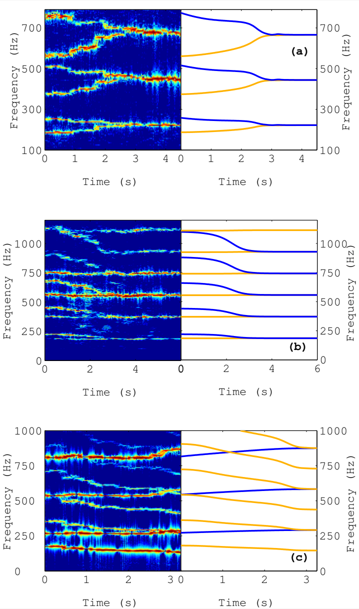
$$\boxed{\frac{d\omega_{i0}}{dt} = \sum_{j\mu} K_{ij}\ I_{j\mu}\ \sin\left(2m\pi\frac{\omega_{j\mu}}{\omega_{i0}}\right) \qquad i(j) = 1, ..., N}$$

$\omega_{j\mu}$: Frecuencia del $\mu-$ésimo armónico del $j-$ésimo individuo

$I_{j\mu}$: Intensidad en la descomposición Fourier

K_{ij} : Acoplamiento $\rightarrow$ 'Cuánto i escucha a j'. En general $K_{ij} \neq K_{ji}$

Modelo de afinación. Ejemplos



(a) Barítono y mezzo soprano.
$m = 1,\ K_{21} = K_{12},\ \frac{\omega_{20}}{\omega_{10}} = 1.$

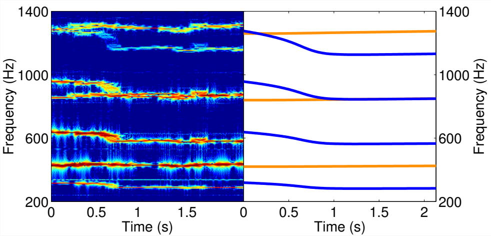
(b) Dos tenores.
$m = 1,\ K_{21} = 25K_{12},\ \frac{\omega_{20}}{\omega_{10}} = 1.$

(c) Barítono y soprano.
$m = 1,\ K_{21} = -K_{12}!!,\ \frac{\omega_{20}}{\omega_{10}} = 2.$

Modelo de afinación. Ejemplos

Consigna: *'Afinar (sensación agradable)'*

Tenor y mezzo soprano.



$m = 2, \; K_{21} = -0{,}15 K_{12}$.

Inicial $\frac{\omega_{20}}{\omega_{10}} = \frac{4}{3}$ (cuarta).

Final $\frac{\omega_{20}}{\omega_{10}} = \frac{3}{2}$ (quinta).

Modelo de afinación. Condición de estabilidad

- Ecuación:

$$\frac{d\omega_{i0}}{dt} = \sum_{j\mu} K_{ij}\ I_{j\mu}\ \sin\left(2m\pi\frac{\omega_{j\mu}}{\omega_{i0}}\right)$$

- Dos cantantes $\rightarrow \omega_{j\mu} = (\mu + 1)\ \omega_{j0}$
- Condición (suficiente): $2m(\mu+1)\frac{\omega_{j\mathbf{0}}}{\omega_{i\mathbf{0}}} = n_{ij}$
- Se llega a:

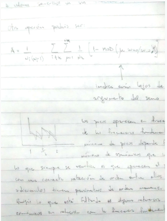
$$\left[K_{21}\sum_{\mu} I_{2\mu}\,(-1)^{(2m\mu/r)} + r^3 K_{12}\sum_{\mu} I_{1\mu}\mu\right] > 0$$

$$r = \frac{\omega_{20}}{\omega_{10}}, \qquad \frac{K_{21}}{K_{12}}:\ \text{parámetro libre}$$

Discusión abierta

Preguntas abiertas

- ¿Qué m elegir?
- ¿Qué pasa con coros? (Hicimos un intento con coros de chicharras)
- Condiciones de estabilidad más generales.
- ¿Cuál es un buen parámetro de orden de afinación?



- ¿Se puede defnir una distancia entre espectros?
- Etc., etc., etc.

¿Preguntas?